A GENERAL CORRELATION FOR HEAT TRANSFER DURING SUBCOOLED BOILING IN PIPES AND ANNULI

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INTRODUCTION

Because of the practical importance of the problem, a considerable amount of research on heat transfer during subcooled boiling flow has been done in recent years, and many techniques for prediction of heat transfer rates have been proposed. Most of these are dimensional equations relating wall superheat to pressure and heat flux or to heat flux alone, and then are intended for only one fluid within a certain range of parameters. Comparatively few attempts have been made at developing general correlations which may be applicable to a wide variety of fluids and operating parameters. The author is aware of only one correlation, that by Rohsenow, which has been compared to a fairly wide variety of data. However, there are severe restrictions on its general use. Thus the need for a general predictive technique seems evident.

The author’s objective was to develop a general correlation to predict heat transfer coefficients during partial and fully developed subcooled boiling with an accuracy comparable to that of single phase correlations which is about ±30%. Judging by the agreement of the proposed correlation with experimental data, the objective appears to have been substantially fulfilled. About 500 data points from 29 data sets (from 18 independent experimental studies) are correlated with a mean deviation of 9.5%, with 97.5% of the data within ±30%. These data include fluids water, R-11, R-12, R-113, ammonia, isopropyl alcohol, n-butyl alcohol, methyl alcohol (methanol), and aqueous solutions of potassium carbonate in horizontal and vertical pipes and annuli. Pipe diameters range from 2.4 to 27.1 mm and pipe materials include stainless steel, copper, nickel, inconel, and glass. Pressures range from 0.1x10^6 to 13.8x10^6 N/m², reduced pressure from 0.005 to 0.96, subcooling from 0 to 153 deg C, heat flux from 0.01x10^6 to 22.9x10^6 W/m², and mass flux from 0.2x10^6 to 87x10^6 kg/h m². Thus the range of parameters covered is quite wide and general applicability appears quite probable.

In the following, the correlation is presented, its development described, and the limits of its applicability explored through data analysis. So that the correlation may be viewed in the proper perspective, brief discussions on some other predictive techniques are also included.

THE CORRELATION

The correlation is expressed by two simple equations applicable in different ranges of subcooling.

\[
\begin{align*}
\text{(low subcooling region)} & \quad \Psi = \Psi_0 \\
\text{(high subcooling region)} & \quad \Psi = \Psi_0 + \frac{\Delta T_{sc}}{\Delta T_{sat}}
\end{align*}
\]

(1)  (2)

The demarcation between high and low subcooling regions is shown later in Fig. 5. Essentially, the low subcooling region corresponds to fully developed boiling.

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while high subcooling region corresponds essentially to partial or local boiling. In the foregoing equation

$$
\psi = \frac{q}{(\Delta T_{SAT} ~ h_L)}
$$

(3)

$$
\psi_0 \text{ is the value of } \psi \text{ at zero subcooling and zero vapor quality and } h_L \text{ is the heat transfer coefficient for all mass flowing as liquid without any boiling. Calculation of } h_L \text{ is discussed in the next section. } \psi_0 \text{ is given by Eq 12 and 13 and is a function of the Boiling number } Bo \text{ defined as:}
$$

$$
Bo = \frac{q}{(G ~ h_{fg})}
$$

(4)

In Eq 4, q is the heat flux, G is the mass flux, and h_{fg} is the latent heat of vaporization. Furthermore, the two phase or boiling heat transfer coefficient h_{TP} is defined as

$$
h_{TP} = \frac{q}{(T_w - T_B)} = \frac{q}{(\Delta T_{SC} + \Delta T_{SAT})}
$$

(5)

Fig. 1 is a graphical representation of the proposed correlation, Eq 1 and 2. It is possible to use this figure directly for design calculations but the mathematical equations will generally be found more convenient.

SINGLE PHASE HEAT TRANSFER

Solution of Eq 1 and 2 requires the calculation of the single-phase heat transfer coefficient h_{L}. This topic is therefore discussed here first. Circular pipes and annular sections are discussed separately.

Circular Pipes

For fully turbulent flow in circular pipes, the best known correlation is the Dittus-Boelter equation:

$$
h_{L} \frac{D}{k} = 0.023(GB/\mu)^{0.8} \text{ Pr}^{0.4}
$$

(6)

All properties in Eq 6 are calculated at the bulk fluid temperature. Single-phase flows are generally fully turbulent at Re_{L} > 10,000 though the transition from laminar to turbulent occurs around Re_{L} = 2,300. In most cases Eq 6 can be used without much error to lower Reynolds numbers, say down to 5,000.

Another well-known correlation for turbulent flows is the Sieder-Tate equation:

$$
h_{L} \frac{D}{k} = 0.027(GB/\mu)^{0.8} \text{ Pr}^{0.33} \left(\frac{\mu}{\mu_w}\right)^{0.14}
$$

(7)

In Eq 7 all properties are calculated at the bulk fluid temperature except \mu_{w} which is calculated at the wall temperature. Predictions of Eq 6 and 7 are virtually the same for moderate values of wall-liquid temperature differences and Prandtl numbers. At high Prandtl numbers and high temperature differences, Eq 7 is somewhat more accurate. However, it requires iterative calculations and is hence cumbersome to use.

All circular pipe data analyzed are for Re_{L} > 10,000 except for 4 data points from the study by Bergles and Rohsenow which are at Re_{L} = 2,300. With so few data points at low Reynolds numbers, the possibility of using another correlation at low Re_{L} could not be explored adequately and Eq 6 was used to calculate h_{L} throughout. It was realized that Eq 7 would have been preferable but it would have involved excessive calculation effort for only a small improvement in accuracy. Finally, in analyzing the data of Krieth and Summerfield for n-butyl alcohol, the constant in Eq 6 was changed from 0.023 to 0.033. The reason was that their single-phase measurements were 40% higher than the predictions of Eq 6 and 7. As the measurements for nonboiling water carried out in the same test section show good agreement with these equations, this deviation cannot be attributed to entrance effects. The most probable cause appears to be that the property data for n-butyl alcohol in this range (31 to 48°C) are erroneous.

Annuli

For turbulent flow through an annulus formed by two concentric circular pipes,
it is generally accepted that heat transfer coefficients can be calculated with Eq 6 or 7 if D is replaced by \( D_e \), the equivalent or effective diameter of the annulus. \( D_e \) is defined as 4 times the flow area divided by the wetted perimeter. Mathematically

\[
D_e = \frac{D_o^2 - D_i^2}{D_o + D_i} = D_o - D_i \quad (8)
\]

For all data examined during this study, substitution of \( D_e \) from Eq 8 into Eq 6 provided excellent agreement with measured single-phase heat transfer coefficients, including those for the 2.2 mm clearance annulus used in some tests of McAdams et al.

Another definition of \( D_e \) which has occasionally been used is 4 times the flow area divided by the heated perimeter. Assuming that the inner pipe is heated and there is no heat transfer from the outer pipe, this definition yields

\[
D_e = \frac{D_o^2 - D_i^2}{D_i} \quad (9)
\]

All single phase heat transfer data known to this author are better correlated using \( D_e \) from Eq 8 than that from Eq 9. In analyzing the boiling heat transfer data, \( D_e \) has been calculated throughout with Eq 8 except in the case of the data of McAdams et al for a 2.2 mm clearance annulus where Eq 9 has been used. The reason for this departure from the usual definition is discussed later.

Single phase data for water and methanol heat transfer from experiments of Colburn et al are shown in Fig. 2. It is noted that for \( \text{Re} > 3,000 \), Eq 6 correlates the data well though the measurements are slightly higher than predictions and would be correlated better with Eq 7. For lower Reynolds numbers, the measurements are markedly higher than Eq 6 and satisfy the equation

\[
\text{Nu} = 0.47 \, \text{Re}_L^{0.44} \, \text{Pr}^{0.4} \quad (10)
\]

It is also noted that in annuli, heat transfer coefficients at low Reynolds numbers cannot be calculated by substituting \( D_e \) in laminar pipe flow equations. Thus the predictions of the Sieder-Tate laminar flow equation are seen to be much lower than measurements.

There does not appear to be any correlation for flow in annuli which has been verified against a wide variety of data. An investigation of single phase heat transfer was not the objective of this study and all data for \( \text{Re}_L < 2,300 \) were from the one data section used by Colburn et al. \( h_L \) at \( \text{Re}_L < 2,300 \) was calculated using Eq 10. It must be stressed that Eq 10 is not being advocated as a general correlation for heat transfer at low Reynolds numbers.

**INCEPTION OF BOILING**

The necessary condition for boiling to occur is that the wall temperature be greater than the saturation temperature of the fluid, i.e.,

\[
\Delta T_{\text{SAT}} = (T_w - T_{\text{SAT}}) > 0 \quad (11)
\]

However, the fulfillment of this condition is not sufficient to ensure boiling. Whether boiling occurs or not appears to depend on many factors among which are heater surface geometry, pressure, heat flux, subcooling, velocity and presence of dissolved gases. If appreciable amounts of dissolved gases are present, most of the evidence such as experiments of McAdams and Hill indicates that bubble nucleation will commence as soon as inequality 11 is satisfied. For pure fluids, however, inception of boiling appears to be largely unpredictable. The problem is discussed in some detail by Griffith who quotes experiments in which, under apparently identical conditions, the wall superheat required to initiate boiling varied from 7 to 42 deg C, a factor of 6.

The problem is highly complex and cannot be adequately discussed here. Several methods of predicting boiling inception are discussed by Coller which may be consulted as a general reference on this subject. In the present data analysis it has been assumed throughout that nucleate boiling starts immediately when \( T_w \) exceeds \( T_{\text{SAT}} \).
SATURATED BOILING AT ZERO QUALITY

The development of this correlation was started by analyzing the experimental data at approximately zero vapor quality and zero subcooling. Data for mean vapor quality less than 1% were also included as it extended the range and variety of data while such small qualities do not significantly affect the heat transfer coefficients as study of Ref 27 will show. The results of this analysis are shown in Fig. 3. It is seen that the data are well correlated by the following two equations.

\[
\begin{align*}
\text{Bo} & > 0.3\times10^{-4}, & \psi_0 &= 230 \text{ Bo}^{0.5} \\
\text{Bo} & < 0.3\times10^{-4}, & \psi_0 &= 1 + 46 \text{ Bo}^{0.5}
\end{align*}
\] (12)

Only 3 data points are outside ±30% of these equations. Most of the data are in fact within ±20%. These data include 11 different liquid-surface combinations and a very wide range of heat flux, mass flux and pressure. Thus it appears that Eq 12 and 13 are not restricted to any particular liquid-surface combination which is the major difficulty with the Rohsenow correlation.

EFFECT OF SUBCOOLING

Experimental studies such as that by Bernath and Begell show that increasing subcooling initially does not affect the wall temperature. With increasing subcooling, a point is reached where wall temperature and hence \(\Delta T_{\text{SAT}}\) begins to decrease. The region in which subcooling has no significant will be called the low subcooling region. It corresponds roughly to the fully developed boiling region. The region in which subcooling affects the wall temperature will be called the high subcooling region and corresponds roughly to the partial or local boiling region. In the low subcooling region, \(\Delta T_{\text{SAT}}\) may be expected to equal that given by Eq 12 and 13. In other words, Eq 1 will apply in the low subcooling region. We will now attempt to define these two subcooling regions quantitatively.

The first attempt is made by plotting \((\psi/\psi_0)\) against \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}}\) as shown in Fig. 4. It appears that the transition occurs around \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} = 2\). For \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} > 2\), the mean through the data may be expressed by the following equation:

\[
\psi/\psi_0 = 0.54 \left(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}}\right)^{-0.88}
\] (14)

A closer look at the data reveals that for low boiling numbers, the transition appears to depend also on Bo. Very few data were available from which the transition point could be determined. Furthermore, the transition point is generally not well defined. The available data are shown in Fig. 5.

Scatter is considerable and data for higher boiling numbers are unavailable. Setting the upper limit for transition at \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} > 2\) based on Fig. 4, and drawing the mean through data, a curve is obtained. Below this curve, Eq 1 applies. Above this curve, subcooling affects the wall temperature and Eq 1 is no longer valid. The proper correlation for this region will be discussed later in this paper.

For \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} < 2\), the transition curve is expressed by the following equation

\[
\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} = 6.3\times10^4 \text{ Bo}^{1.25}
\] (15)

Thus the correlation for the low subcooling region has been determined and the demarcation between high and low subcooling regions established. The development of the correlation for the high subcooling region is now undertaken.

High Subcooling Region

From Fig. 4 it would appear that for \(\Delta T_{\text{SC}}/\Delta T_{\text{SAT}} > 2\), Eq 14 may be suitable. However it was found to be highly erratic. The reason becomes evident when it is transformed into the following form.
\[ \Delta T_{\text{SAT}} = \left[ \frac{a}{0.54 h_L \psi_0} \right] 8.33 (\Delta T_{\text{SC}}) - 7.83 \]  

In Eq 16, \( \Delta T_{\text{SAT}} \) is a very strong function of \( h_L \), small variations in \( h_L \) affecting \( \Delta T_{\text{SAT}} \) drastically. No method of determining \( h_L \) exactly is known. Furthermore, such a strong dependence is not substantiated by experimental evidence. Hence another equation for this region was sought.

One possible approach could be to assume that the total heat flux removed is the sum of heat flux removed by single phase convection and the heat flux removed by nucleate boiling, i.e.,

\[ q = q_{\text{SPC}} + q_{\text{NB}} \]  

The validity of Eq 17 cannot be investigated unless the method for calculating the two components is specified. Rohsenow\textsuperscript{1} assumed that \( q_{\text{NB}} \) can be estimated by his pool boiling correlation, \( q_{\text{SPC}} \) being calculated with a correlation such as Eq 6. However, Bergles and Rohsenow\textsuperscript{15} conducted experiments which showed that this assumption is not correct. They recommended that \( q_{\text{NB}} \) be estimated from data on fully developed flow boiling. This approach has been followed here.

At zero subcooling, the definition of \( \psi_0 \) yields the following relation:

\[ q = h_L(T_w - T_{\text{SAT}}) + h_L(\psi_0 - 1)(T_w - T_{\text{SAT}}) \]  

Comparing Eq 17 and 18, it is noted that

\[ q_{\text{NB}} = h_L(\psi_0 - 1)(T_w - T_{\text{SAT}}) \]  

As the value of \( \psi_0 \) has been determined from the data on flow boiling at zero subcooling, which may be considered to be fully developed boiling, Eq 19 is according to the recommendation of Bergles and Rohsenow mentioned earlier.

It is now postulated, subject to experimental verification that Eq 19 continues to hold in the high subcooling region and that \( q_{\text{SPC}} \) can be calculated by the following equation:

\[ q_{\text{SPC}} = h_L(T_w - T_B) \]  

where \( h_L \) is calculated by an equation suitable for single phase heat transfer under the prevalent conditions. Substituting from Eq 19 and 20 into Eq 17

\[ q = h_L (T_w - T_B) + h_L(\psi_0 - 1)(T_w - T_{\text{SAT}}) \]  

This can be rearranged into the following more convenient form:

\[ \psi = q/(\Delta T_{\text{SAT}} h_L) = \psi_0 + \Delta T_{\text{SC}}/\Delta T_{\text{SAT}} \]  

Eq 22 is the same as Eq 2 and it is shown later that it is in reasonable agreement with experimental data in the high subcooling region as defined by Fig. 5.

**DATA ANALYSIS**

The salient features of the data that have been used to develop and evaluate the proposed correlation are listed in Table 1. The complete range of parameters which could possibly be significant, covered by the data analysis, is given in Table 2.\textsuperscript{14} Data of Noel,\textsuperscript{14} Glkrk and Rohsenow,\textsuperscript{15} Bergles and Rohsenow,\textsuperscript{5} Gouse and Coumou,\textsuperscript{16} Hodgeson,\textsuperscript{1} McAdams,\textsuperscript{2} and Naitoh et al\textsuperscript{20} have been extracted from graphical representations in these references. Other data are from tabulations. Where the references provided a large amount of data points, data samples representative of the range of experimentation were picked out at random. This was done to keep the calculation effort within reasonable limits.

In the tests of Colburn et al,\textsuperscript{3} heating was done with condensing steam and the heat flux was not constant along the length. All other data listed in Table 1 were obtained by electric heating providing essentially constant heat flux.
In all experiments on annular test sections, heat was applied to the inner pipe of the annulus and the outer pipe was insulated. Finally, all refrigerants used (R-11, R-12, R-113, ammonia) were completely free of oil.

The properties of water, R-11, R-12, R-113, and ammonia were taken from Ref 21 though some high pressure properties had to be taken from other sources. Properties of 35% and 50% potassium carbonate were taken from the table in the paper by Piret and Ibensen. Properties of other fluids have been taken from Ref 22. Viscosity, thermal conductivity, and specific heat have been estimated at the bulk fluid temperature. Latent heat has been estimated at the saturation temperature.

**RESULTS OF DATA ANALYSIS**

Fig. 6 and 7 show the comparison of some representative measured heat transfer coefficients with the predictions of the correlation. It is seen that measured values range from as low as 700 to as high as 230,000 W/m²·deg C and in most cases agreement is within ±30%. The results of such comparisons are summarized in Table 3. Data points from all sources are correlated with a mean deviation of 9.5%, giving equal weight to each data point. Over 97% of the data points are correlated within ±30%. The deviation of each data point has been calculated by the following equation:

\[
\text{Deviation} = \frac{\text{Predicted Value} - \text{Measured Value}}{\text{Measured Value}}
\]

The mean deviation of each data set has been calculated as the sum of absolute values of individual deviations (ignoring positive or negative signs) divided by the number of data points in that set.

**DISCUSSION OF RESULTS**

The results of the foregoing data analysis indicate that the objective of developing a general correlation for subcooled boiling with an accuracy comparable to that of single-phase heat transfer correlations has been substantially fulfilled. Over 97% of the data are within ±30% and in fact most of the data points are within ±20%. The accuracy of available single-phase correlations is only slightly better. Another way to evaluate this correlation will be to use Eq 1 and 2 to calculate \( h_L \) from the boiling heat transfer data. In almost all such cases, \( h_L \) so calculated is found to agree with predictions of single-phase correlations within ±30%. While a better accuracy is desirable, whether or not a substantially more accurate correlation can be found is debatable. Firstly, single-phase convection clearly plays an important part in subcooled boiling heat transfer and hence estimation of boiling heat transfer generally requires estimation of single-phase heat transfer coefficients. Thus the accuracy of single-phase correlations limits the accuracy of boiling heat transfer correlations. Secondly, detailed geometry of heating surface is rarely known. Even if it is known, the behavior of cavities has generally been found unpredictable. Any particular cavity may or may not be active under any particular set of conditions such as pressure, temperature, heat flux and mass flux. An associated phenomena is the hysteresis effect so noticeable in the data of Hodgson. If a particular set of conditions is reached by decreasing heat flux or increasing subcooling, higher heat transfer coefficients are obtained than if the same conditions were reached by increasing heat flux or decreasing subcooling. Furthermore, presence of substantial amounts of dissolved gases is known to cause early inception of boiling and generally higher heat transfer coefficients; however, the quantitative relation between concentration of dissolved gases and heat transfer phenomena is not known. A highly accurate correlation will have to consider detailed surface geometry, concentration of dissolved gases, and the process through which these prevalent parameters are reached. Development of such a correlation requires much more basic information than is presently available and would have to await more research into the boiling phenomena.

While all other data analyzed are for constant heat flux conditions, those of Colburn et al. are for variable heat flux as heating was done by condensing steam. Satisfactory correlation of these data indicates that the proposed correlation is
applicable to both variable and constant heat flux conditions. Furthermore, data for almost all commonly used commercial plain pipes are included and satisfactorily correlated. Hence it appears that within the designated accuracy, the correlation is applicable to all commercial plain pipes. The accuracy of the correlation seems independent of pipe diameter, pressure, heat flux, mass flux, and Boiling number. However, the applicability at low Reynolds numbers remains open to question and is discussed later.

From Table 3 and Fig. 6 and 7, it is noted that the only data set that shows excessive deviation is that of Piret and Iselin for carbon tetrachloride. The properties of this fluid are not too different from those of halocarbon refrigerants (Freons). Parameters such as heat and mass flux, ReL, Bo are within the range covered for other fluids. Hence no definite explanation for this deviation can be offered and it could be that some phenomena are involved which this correlation does not account for. However, when only 1 set of 3 data points out of 29 data sets show large deviation, it is more reasonable to consider the possibility of experimental error or presence of scale on the pipe surface.

Effective Diameter of Annulus

The question of effective diameter for annuli may now be discussed. In Fig. 8, predicted and measured heat transfer coefficients in annuli are compared. All predictions are based on Dg calculated by Eq 8, i.e., Dg is based on wetted perimeter. It is noted that out of the 6 data sets, 5 are well correlated while one data set is too low. This set is from the annulus with the smallest clearance, 2.2 mm. The next smallest annulus had a clearance of 4.3 mm. Hence a preliminary conclusion that can be drawn is that with decreasing clearance, a point is reached where enhancement in heat transfer is lower than that predicted by Eq 1 and 2. If this hypothesis is true, this transition point is at a clearance somewhere between 4.3 and 2.2 mm. If the data for the 2.2 mm clearance are analyzed with Dg calculated by Eq 9, i.e., based on heated perimeter instead of wetted perimeter, much better correlation is obtained. It is interesting to note that both Chintal and Shah faced the same difficulty in developing their correlations for saturated boiling while analyzing the data of Bemmel et al. in a narrow annulus. The clearance in that annulus was 3.1 mm. If Dg was based on wetted perimeter, prediction of both correlations was too high. Satisfactory correlation was obtained when Dg was based on heated perimeter. The resolution of this problem requires analysis of much more data from varied sources. For the present, we can merely report the observation that the available data for saturated and subcooled boiling in annuli with clearances between 3.1 and 2.2 mm are better correlated with Dg based on heated perimeter. Data for clearance 4.3 mm and higher are better correlated with Dg calculated in the usual way, i.e., based on wetted perimeter. The best recommendation that the author can make presently is given in the Appendix to Ref. 27.

Effect of Reynolds Number

All data for pipes were at Reynolds number higher than 10,000 except for 4 data points at ReL = 2,300 from the study by Bergles and Rohsenow. The predictions for them were somewhat high when hL was calculated by Eq 6. If hL was calculated by a laminar flow equation, the predictions were somewhat low. Furthermore, these authors report that conditions during these tests were very unstable and wall temperatures varied greatly. Hence available data for pipes do not permit any definite conclusion as to the applicability of the correlation at ReL less than 10,000.

The Reynolds numbers in the data of Colburn et al. for flow in annulus range from 50,000 to 1,400. These indicate that the proposed correlation remains valid at lower Reynolds numbers if hL is calculated by an equation applicable at such Reynolds numbers. However, the data are too few to permit complete confidence in this hypothesis. Analysis of much more varied data at low Reynolds numbers is required to reach a definite conclusion.

DESIGN RECOMMENDATIONS

Due to insufficient data at lower values, this correlation is presently recommended only for ReL greater than 10,000. Either of Eq 6 or 7 may be used to
calculate \( h_L \). Eq 7 is to be preferred as it is generally more accurate.

For \( \text{Re}_L \) less than 10,000, some other predictive technique may be sought. In the case this correlation has to be used due to unavailability of any other reliable method, \( h_L \) should be calculated with some correlation suitable for the prevalent Reynolds number. Generally Eq 6 and 7 can be used without much error at \( \text{Re}_L \) as low as 4,000.

Application to metallic fluids is not recommended. If this correlation is used as a last resort, \( h_L \) should be calculated with an equation suitable for metallic fluids.

This correlation is not recommended for annuli with clearance less than 4 mm. In case it has to be used, \( D_e \) should be based on heated perimeter. For annuli with clearance greater than 4 mm, \( D_e \) is to be based on wetted perimeter.

**Calculation Procedure**

To clarify the use of this correlation, the procedure for solving a typical design problem is outlined. Pipe diameter, heat flux, mass flux, pressure, and subcooling are known. Heat transfer coefficient has to be calculated. The following steps are suggested:

1. Calculate \( h_L \).
2. Calculate the wall temperature assuming no boiling. If \( T_w \) is found lower than \( T_{SAT} \), no boiling can occur and no further calculations are needed.
3. If Step 2 shows \((T_w - T_{SAT}) > 0\), calculate \((\Delta T_{SAT})_p\) using a reliable predictive technique. If \((\Delta T_{SAT})_p\) is greater than \( \Delta T_{SAT} \) calculated in Step 2, no boiling occurs and \( h_{TP} = h_L \). If predictive techniques not reliable or liquid contains substantial amounts of dissolved gases, assume boiling starts immediately when \( \Delta T_{SAT} > 0 \) and proceed to the next step.
4. Use Eq 1 to calculate \( \Delta T_{SAT} \).
5. Calculate \((\Delta T_{SG}/\Delta T_{SAT})\).
6. Check in Fig. 5 whether Eq 1 is applicable. If not, use Eq 2 to calculate \( \Delta T_{SG} \).
7. Use Eq 5 to calculate \( h_{TP} \).

In many cases heat flux is not known. Instead, the temperature and heat transfer coefficient of heating medium is known. Iterative calculations with assumed values of heat flux are then required until the imposed boundary conditions are satisfied.

A few words regarding the influence of lubricating oil in refrigerant evaporators are in order. For ammonia which is insoluble in oil, experiments of Shah,24,25 indicate that insulating oil films are likely to form. These two papers may be consulted for estimating resistance of these oil films. For freon refrigerants, which are soluble in oil, experimental evidence is apparently conflicting. To deal with this topic adequately, detailed discussions will be required. Hence no recommendations for freon-oil mixtures are made.

**OTHER PREDICTIVE TECHNIQUES**

To evaluate other predictive techniques in detail was outside the scope of this study. However, some discussions on this subject are necessary so that the correlation proposed here may be viewed in the proper perspective.

The vast majority of available correlations are dimensional equations intended for only one particular fluid in a limited range of parameters. Among these dimensional equations, the majority expresses \( \Delta T_{SAT} \) as a function of \( q \) alone. A typical example is the correlation of McAdams et al.22. As experiments clearly show that pressure is also an important parameter, the applicability range of such equations is very limited. More sophisticated dimensional correlations also include pressure as a parameter. A notable example of such correlations is that proposed by Jens and Lottes26 for water which may be written as

\[
\Delta T_{SAT} = 25 q^{0.25} e^{-p/62}
\]  

(23)
where $\Delta T_{SAT}$ is in deg C, $q$ is in kW/m², and $p$ is in bar. This equation has found wide acceptance for practical designs. It was compared to some of the boiling water data analyzed here. The accuracy of predictions was found comparable to Eq 1 and 2. An objection that may be raised against the Jens and Lottes equation and other such equations is that they assert that subcooling has no influence while experiments such as those of Hodgson, Bernath and Begell show that in high subcooling region, it is not true. However, from a practical standpoint, the error involved is not too much as at high subcooling, $\Delta T_{SAT}$ is only a fraction of $\Delta T_B$.

Applicability to more than one fluid and wide operating conditions may be expected from dimensionless equations. Very few such equations have been proposed. Hodgson has presented an equation which correlates his own data, those of Clark and Rohsenow for water, and Noel for ammonia, with an accuracy of ±30%. Hence it is possible that the Hodgson correlation may be generally applicable. It may be mentioned that the present correlation shows better agreement with the same data.

The best known dimensionless correlation is that of Rohsenow. However, its generality is restricted by the fact that it involves a multiplier $C_{sf}$ which is different for each liquid-surface combination. Reported values of $C_{sf}$ vary from 0.0027 to 0.02. No method for predicting $C_{sf}$ is available and it has to be determined from experimental data obtained with the same surface-liquid combination. As the number of liquids is virtually limitless, the utility of this correlation is limited. Furthermore, as was mentioned earlier, the experiments of Bergles and Rohsenow put the validity of this method in doubt. The Rohsenow correlation has been compared to data from Ref 16, 10, 15, 8, and 9. With the exception of the carbon tetrachloride data from Ref 16, the accuracy obtained appears comparable with that of the present correlation.

From the foregoing, it appears that the correlation proposed here is generally preferable to other predictive techniques. However, it is possible that some better correlation may be available in the vast literature on this subject which has not come to this author's notice.

CONCLUDING REMARK

The correlation presented in this paper appears to be generally applicable for predicting heat transfer coefficients in fully developed and local subcooled boiling in pipes with an accuracy of ±30%. Where such accuracy is acceptable, it is suggested that this correlation be given serious consideration. It must be stressed that this correlation is not being advocated as a perfect solution. The phenomena involved are so complex that the perfect solution may not be forthcoming for some considerable time.

Much more data analysis is required at Reynolds numbers lower than 10,000, specially lower than 2,300. It is possible that this correlation may also be applicable to boiling metals and hence analysis of such data is desirable. Analysis of data from narrow annuli of clearance less than 4 mm, annuli heated from both sides, and annuli heated from the outer pipe is also needed. Finally, several flow channel geometries beside pipes and annuli are also of practical interest. An important case is the flow parallel to heated rod bundle enclosed in a cylindrical shell, which occurs in pressurized water nuclear reactors. While no experimental data for such a case were available, some calculations for such reactors done using other correlations were available and showed fair agreement with the predictions of this correlation. Hence application to flow channels other than pipes and annuli is probable and worth investigating through data analysis.

NOMENCLATURE

\begin{align*}
A & \quad \text{Cross-sectional area} = \pi D^2/4 \\
Bo & \quad \text{Boiling number, } q/(G h_{fg}) \\
C_{sf} & \quad \text{Multiplier in Rohsenow correlation}
\end{align*}
D  Inside diameter of pipe
D_e  Effective or equivalent diameter of annulus
D_i  Inner diameter of annulus
D_o  Outer diameter of annulus
G  Mass flux, W/A
h_L  Heat transfer coefficient for nonboiling flow
h_{fg}  Latent heat of vaporization
h_{TP}  Two-phase or boiling heat transfer coefficient
k  Thermal conductivity of liquid
Nu  Nusselt number, h_L D/k
L  Length of pipe
Pr  Prandtl number of liquid
P_{re}  Reduced pressure, i.e., actual pressure divided by critical pressure
p  Absolute pressure of fluid
q  Total heat flux
q_{NB}  Heat flux removed by nucleate boiling
q_{SPC}  Heat removed by single-phase convection
Re_L  Reynolds number of liquid, GD/μ
T_B  Bulk fluid temperature, i.e., temperature measured by a thermal sensor in the fluid; essentially the mixing-cup temperature
T_{SAT}  Saturation temperature
T_w  Wall temperature
ΔT_B  (T_w - T_B)
ΔT_{SAT}  Wall superheat, (T_w - T_{SAT})
ΔT_{SC}  (T_{SAT} - T_B = (ΔT_B - ΔT_{SAT})
(ΔT_{SAT})_{Bi}  Wall superheat required to initiate boiling
W  Total mass flow rate, vapor plus liquid
x  Thermodynamic vapor quality
ψ  q/(ΔT_{SAT} h_L)
ψ_o  Value of ψ at T_{SC} = 0, and x = 0. Calculated by Eq 12 and 13
μ  Dynamic viscosity of liquid
μ_w  Dynamic viscosity of liquid at wall temperature

REFERENCES


ACKNOWLEDGEMENT

The author thanks Dr. M.B. Noel for providing a copy of his paper (Ref 14).
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>TEST SECTION</th>
<th>FLUID WALL</th>
<th>$\Delta T_{SC}$</th>
<th>$\Delta T_{SAT}$</th>
<th>$\rho_{L} \cdot x_{L} \cdot 10^{-3}$</th>
<th>$P_e \cdot 10^{4}$</th>
<th>$\Delta T_{SC}/\Delta T_{SAT}$</th>
</tr>
</thead>
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<tr>
<td>Colburn et al.</td>
<td>Vert. ann. 50.8 water copper</td>
<td>0.11</td>
<td>101</td>
<td>2</td>
<td>3.0</td>
<td>0.06</td>
<td>1.4</td>
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<td>mm OD, 42.2 mm ID</td>
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<td>56</td>
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<td>0.21</td>
<td>23.2</td>
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<td>Noel</td>
<td>Vert. pipes 5.9 ammonium nla</td>
<td>1.17</td>
<td>30</td>
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<td>Riedle &amp; Ferucupile</td>
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<td>0.05</td>
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<td>0</td>
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<td>0.01</td>
<td>31.2</td>
<td>0.4</td>
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<td>0.60</td>
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<tr>
<td>ID pipe</td>
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<td>335</td>
<td>153</td>
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<td>9.45</td>
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<td>2.4 mm ID</td>
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<td>Hor. 10.9 mm R-113 glass</td>
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<td>47</td>
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<td>0.01</td>
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<td>18.4</td>
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<td>Pipe 7.9 mm water SS</td>
<td>0.26</td>
<td>129</td>
<td>99</td>
<td>4.7</td>
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<td></td>
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<td>133</td>
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<td>3.04</td>
<td>32.0</td>
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<td>Dougall &amp; Panian</td>
<td>Vert. ann. 19 R-113 SS</td>
<td>1.0</td>
<td>179</td>
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<td>5.6</td>
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<td>1.75</td>
<td>205</td>
<td>50</td>
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<td>133.9</td>
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<td>Noda &amp; et al.</td>
<td>Vert. annuli 6.3 Water SS</td>
<td>0.21</td>
<td>121</td>
<td>28</td>
<td>1.0</td>
<td>0.20</td>
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<td>mm ID, 19.5 &amp; 18.5</td>
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<td>0.62</td>
<td>160</td>
<td>48</td>
<td>3.9</td>
<td>4.43</td>
<td>79.8</td>
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<td>0.41</td>
<td>145</td>
<td>11</td>
<td>12.1</td>
<td>1.83</td>
<td>32.6</td>
<td>2.5</td>
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<tr>
<td>Id, 10.7 mm OD</td>
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<td>0.41</td>
<td>145</td>
<td>83</td>
<td>12.1</td>
<td>4.41</td>
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<td>Muma</td>
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<td>135</td>
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<td>1.2</td>
<td>0.16</td>
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<td>7.98</td>
<td>98.4</td>
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<td>100</td>
<td>0</td>
<td>1.41</td>
<td>0.02</td>
<td>37.6</td>
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<td>mm ID</td>
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<td>0.10</td>
<td>117</td>
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<td>0.02</td>
<td>35.4</td>
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<td>117</td>
<td>0</td>
<td>2.0</td>
<td>0.02</td>
<td>35.4</td>
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<td>alcohol</td>
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<td>0.10</td>
<td>117</td>
<td>0</td>
<td>2.0</td>
<td>0.02</td>
<td>35.4</td>
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<td>isopropl copper</td>
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<td>0.10</td>
<td>82</td>
<td>0</td>
<td>1.9</td>
<td>0.01</td>
<td>28.3</td>
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<td>alcohol</td>
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<td>0.10</td>
<td>82</td>
<td>0</td>
<td>2.0</td>
<td>0.01</td>
<td>28.3</td>
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<td>CO2 copper</td>
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<td>0.10</td>
<td>77</td>
<td>0</td>
<td>1.2</td>
<td>0.01</td>
<td>19.5</td>
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<td>35% H2CO3 copper</td>
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<td>0.10</td>
<td>106</td>
<td>0</td>
<td>1.8</td>
<td>0.02</td>
<td>16.1</td>
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<td>50% H2CO3 copper</td>
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<td>114</td>
<td>0</td>
<td>2.3</td>
<td>0.02</td>
<td>12.2</td>
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<tr>
<td>Naitoh et al.</td>
<td>Vertical Helix water SS</td>
<td>16.8</td>
<td>354</td>
<td>0</td>
<td>4.5</td>
<td>0.11</td>
<td>287.4</td>
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<td>pipe ID 16.5 mm</td>
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<td></td>
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**TABLE 1**

Salient Features of Experimental Data Analyzed to Evaluate the Correlation
### TABLE 2

Complete Range of Parameters Over Which the Correlation Has Been Verified

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
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<tbody>
<tr>
<td>Pipe ID, mm</td>
<td>2.4 to 27.1</td>
</tr>
<tr>
<td>Annulus clearance, mm</td>
<td>6.6 to 4.3</td>
</tr>
<tr>
<td>Heating surface</td>
<td>Copper, stainless steel, nickel, inconel, glass</td>
</tr>
<tr>
<td>Pressure, (N/m^2\times10^{-6})</td>
<td>0.1 to 13.8</td>
</tr>
<tr>
<td>Reduced pressure</td>
<td>0.005 to 0.76</td>
</tr>
<tr>
<td>(T_{SAT}, ^\circ C)</td>
<td>23 to 354</td>
</tr>
<tr>
<td>(\Delta T_{SC}, \deg C)</td>
<td>0 to 153</td>
</tr>
<tr>
<td>(\Delta T_{SC}/\Delta T_{SAT})</td>
<td>0 to 240</td>
</tr>
<tr>
<td>(G\times10^{-6}, \text{kg/m}^2\text{h})</td>
<td>0.2 to 0.87</td>
</tr>
<tr>
<td>(q\times10^{-6}, \text{W/m}^2)</td>
<td>0.01 to 22.9</td>
</tr>
<tr>
<td>Boiling point, (x\times10^{-3})</td>
<td>0.1 to 54</td>
</tr>
<tr>
<td>Oil content in refrigerants</td>
<td>None at all</td>
</tr>
<tr>
<td>Pr</td>
<td>0.8 to 35</td>
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### TABLE 3

Summary of Results of Comparison of the Proposed Correlation with Experimental Data

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<thead>
<tr>
<th>Source</th>
<th>Fluid</th>
<th>Total No. of Data Points</th>
<th>No. of Data Points with Dev. &gt; ±30%</th>
<th>Mean Deviation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riedle and Percuiple(^13)</td>
<td>R-11</td>
<td>40</td>
<td>1</td>
<td>14.0</td>
</tr>
<tr>
<td>(both diameters)</td>
<td>R-113</td>
<td>38</td>
<td>1</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>R-42</td>
<td>31</td>
<td>1</td>
<td>14.0</td>
</tr>
<tr>
<td>Clark and Rohsenow(^10)</td>
<td>Water</td>
<td>15</td>
<td>0</td>
<td>3.4</td>
</tr>
<tr>
<td>Krieth and Summerfield(^8)</td>
<td>Water</td>
<td>44</td>
<td>0</td>
<td>6.6</td>
</tr>
<tr>
<td>Krieth and Summerfield(^9)</td>
<td>n-butyl alcohol</td>
<td>42</td>
<td>0</td>
<td>3.2</td>
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<tr>
<td>Bergles and Rohsenow(^15)</td>
<td>Water</td>
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<td>0</td>
<td>8.6</td>
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<tr>
<td>Gouse and Coumou(^19)</td>
<td>R-113</td>
<td>5</td>
<td>0</td>
<td>8.2</td>
</tr>
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<td>Hodgson(^12)</td>
<td>Water</td>
<td>58</td>
<td>2</td>
<td>11.4</td>
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<td>Pappel(^7)</td>
<td>Water</td>
<td>13</td>
<td>0</td>
<td>3.1</td>
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<td>Dougall and Fanian(^11)</td>
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<td>39</td>
<td>0</td>
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<tr>
<td>Colburn et al(^3)</td>
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<td>0</td>
<td>8.0</td>
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<tr>
<td></td>
<td>Water</td>
<td>15</td>
<td>0</td>
<td>6.3</td>
</tr>
<tr>
<td>Noel(^14)</td>
<td>Ammonia</td>
<td>27</td>
<td>2</td>
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<tr>
<td>McAdams et al(^2)</td>
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<td>27</td>
<td>2</td>
<td>11.0</td>
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<tr>
<td>clearance, mm</td>
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<td>Water</td>
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<td>3</td>
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<td></td>
<td>6.1</td>
<td>Water</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6.6</td>
<td>Water</td>
<td>6</td>
<td>0</td>
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<td>Piret and Isbin(^16)</td>
<td>Isopropyl Alcohol</td>
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<td>n-butyl Alcohol</td>
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<td>0</td>
<td>6.2</td>
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<td></td>
<td>CCl(_4)</td>
<td>3</td>
<td>3</td>
<td>62.1</td>
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<tr>
<td></td>
<td>32% solution (K_2CO_3)</td>
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<td>0</td>
<td>7.0</td>
</tr>
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<td>50% solution (K_2CO_3)</td>
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<td>Munn(^17)</td>
<td>Water</td>
<td>29</td>
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<td>10.0</td>
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<td>Water</td>
<td>1</td>
<td>0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Total number of data points = 498
Number of data points exceeding ±30% = 13
Mean deviation, percent
giving equal weight to each data point = 9.5
giving equal weight to each data set = 11.1
Percent of data points within ±30% = 97.2
Fig. 1 Graphical representation of the proposed correlation, Eq (1) and (2)

Fig. 2 Data of Colburn et al. for non-boiling flow through an annulus. Heat applied to inner pipe.
Fig. 3 Analysis of boiling heat transfer data at zero subcooling and approximately zero vapor quality.

Fig. 4 First approach for demarcation between low and high subcooling regimes.

Fig. 5 Final demarcation between high and low subcooling regions.
Fig. 6 Measured boiling heat transfer coefficients compared with the proposed correlation, Eq (1) and (2)

Fig. 7 Comparison of measured boiling heat transfer coefficients with predictions of the proposed correlation, Eq (1) and (2)

Fig. 8 Boiling heat transfer coefficients measured in annulii with different values of clearance. Effective diameter based on wetted perimeter throughout.