CHAPTER 11
PREDICTION OF HEAT TRANSFER DURING FORCED-CONVECTION SUBCOOLED BOILING

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INTRODUCTION

This paper is concerned with predicting heat transfer during forced-convection subcooled boiling prior to boiling crisis. Prediction of boiling crisis and heat transfer following boiling crisis are not covered.

In the following, some predictive techniques are given for calculating inception of boiling and heat transfer coefficients for flow in channels and during flow across cylinders. These
predictive techniques are to be considered applicable only to single-component, nonmetallic, noncryogenic fluids, except where specifically noted otherwise. Separate sections of this paper have been devoted to metallic, cryogenic, and multicomponent fluids. These sections should be consulted for these special fluids.

It should be noted that subcooled boiling is also known as surface boiling.

INCEPTION OR ONSET OF NUCLEATE BOILING

Heat transfer due to convection on a wall is given by:

\[ \dot{q}_w = h_w \cdot \dot{T}_w \]

where \( h_w \) is the heat transfer coefficient due to single-phase convection.

The necessary condition for boiling to start is that \( \dot{T}_w > \dot{T}_{sat} \). However, this condition is not sufficient. In many cases, wall temperature can far exceed the saturation temperature without any boiling. For many practical problems, prediction of the onset of boiling is important.

Bergles and Rohsenow [1] presented an analytical model which has found wide acceptance. They postulated that a bubble of radius \( r \) can grow only if the temperature of the liquid at a distance \( r \) from the heating surface is equal to or greater than the temperature of vapor within the bubble. It was assumed that the heating surface contains cavities of the radius required by this condition. They further assumed that heat transfer through the liquid layer occurs by steady-state conduction. The equations resulting from this model were solved graphically and the following empirical equation fitted to the results:

\[ q_{on} = 15.6 \cdot \dot{q}^{0.4} \cdot (\dot{T}_{sat})^{0.38} \cdot h_w^{0.1} \cdot p^{0.38} \cdot T^{0.1} \]

The subscript \( on \) indicates incipient boiling. \( \dot{q} \) is in Btu/hr-ft², \( p \) is in lb/in², and \( \dot{T}_{sat} \) in °F. This equation covers the range \( p = 1 \) to 138 bars.

Davis and Anderson [2] solved the Bergles-Rohsenow equations analytically by making some simplifying assumptions. Their final result was

\[ q_{on} = \frac{k}{8\sigma T_{sat}} \cdot (\dot{T}_{sat})^{1/2} \]

Equation 3 agrees with Equation 2 as well as with the data of Bergles and Rohsenow [1] and Rohsenow and Clark [3], both for water at high pressure. However, the wall superheat was greatly underpredicted for the data of Backhurst [4] for benzene and Matzournis [5] for water, both at atmospheric pressure.

Frost and Drakovic [6] also used the Bergles-Rohsenow model but arbitrarily included the Prandtl number to arrive at the following formula:

\[ q_{on} = \frac{k}{8\sigma T_{sat} \sqrt{Pr}} \cdot (\dot{T}_{sat})^{1/2} \]

Frost and Drakovic consider Equation 4 to be applicable to both forced- and natural-convection systems. The correlation was compared to data for water, hydrogen, freon, acetone, nitrogen, oxygen, neon, ethanol, n-pentane, carbon tetrachloride, benzene, and kerosene. Most of these data were for natural convection boiling (pool boiling).

If the heat flux is the independent variable, Equation 4 directly yields the \( \dot{T}_{sat} \) at onset of boiling. If \( \dot{T}_{sat} \) is the independent variable, Equation 4 has to be solved simultaneously with Equation 3. For forced-convection systems, \( h_w \) is calculated by a forced-convection correlation. For pool boiling, \( h_w \) is to be calculated by a correlation for natural convection. If the liquid contains dissolved gases, onset of boiling occurs at superheats lower than for liquids free of dissolved gases. This was demonstrated experimentally by McAdams et al.
only to single-component fluids, noted otherwise. Separate studies involving single-component and multicomponent fluids led to the conclusion that surface boiling.

\[ \text{Forced-Convection Subcooled Boiling} \]

Heat transfer coefficients during subcooled boiling are often higher with decreasing heat flux than with increasing heat flux. At a constant heat flux, heat transfer coefficients are often higher with increasing subcooling than with decreasing subcooling. This phenomenon is known as hysteresis. The explanation is that with decreasing heat flux, the active bubble nucleation sites tend to remain active at lower heat flux as compared to the case of increasing heat flux. Similar is the situation with increasing subcooling as compared to decreasing subcooling.

Noncryogenic fluids exhibit no hysteresis under saturated conditions. Under subcooled conditions it occurs in many cases but not always. Hysteresis usually increases with increasing subcooling and heat flux and decreases with increasing mass flux. See for example the data of Hodgson [8] for water boiling in a tube.

No method for predicting the occurrence of hysteresis or its magnitude is available. For noncryogenic fluids, the wall temperature variations caused by hysteresis generally do not affect heat transfer coefficients by more than 20% to 30% which is within the accuracy of most predictive techniques.

Heat Transfer During Boiling in Channels

Subcooled boiling in channels can be divided into two regimes, namely the "high subcooling" and the "low subcooling" regimes. In the high subcooling regime, bubble nucleation occurs only on a portion of surface, void fraction is low, bubbles grow and collapse while attached to the wall, and the wall temperature rises with decreasing subcooling. In the low subcooling regime, bubble nucleation occurs over almost the entire surface, void fraction is high, bubbles detach from the wall and are carried in the liquid core, and wall temperature remains constant with decreasing subcooling. The high subcooling regime is also known as the partial boiling regime and the low subcooling regime is also known as the fully developed boiling regime.

Some techniques for prediction of heat transfer in tubes and annuli are now presented. The most verified predictive technique is that of Shah [9, 10] which is discussed first.

Shah Correlation

The correlation of Shah was first presented in Reference 9. Further verification with data for annuli was given in Reference 10. This correlation uses the dimensionless parameters \( B_\theta \), \( \Psi \), and \( \Delta T_{\text{sat}}/\Delta T_{n.s.} \). The first two are defined as:

\[ B_\theta = \frac{q}{(G_{\text{th}})} \]
\[ \Psi = \frac{q}{(\Delta T_{\text{sat}})_{\text{th}}} \]

\( h_{\text{th}} \) is the heat transfer coefficient for single-phase liquid flow.

The value of \( \Psi \) at zero subcooling, \( \Psi_0 \), is given by the following relation:

\[ \Psi_0 = 230 B_\theta \]

If Equation 7 predicts \( \Psi_0 < 1 \), use \( \Psi_0 = 1 \).
HEAT TRANSFER MECHANISMS

In the high subcooling regime, it is assumed that the total heat flux is the sum of heat flux due to single-phase convection and the heat flux to nucleate boiling. Thus:

\[ q = q_{con} + q_{nob} \]  
(8)

At zero subcooling, the definition of \( \psi \) yields the following equation:

\[ q = h_c(T_w - T_{sat}) + h_c(\psi - 1) (T_w - T_{sat}) \]  
(9)

Comparison of Equations 8 and 9 shows that,

\[ q_{nob} = h_c(\psi - 1) (T_w - T_{sat}) \]  
(10)

Thus for the high subcooling regime, the following relation results:

\[ \psi = \psi_0 + \frac{\Delta T_{sc}}{\Delta T_{sat}} \]  
(11)

For the low subcooling regime,

\[ \psi = \psi_0 \]  
(12)

The demarcation between high and low subcooling regimes was found by trial and error and is as follows. When \( \frac{\Delta T_{sc}}{\Delta T_{sat}} > 2 \), the regime is high subcooling. The regime is also that of high subcooling when,

\[ \frac{\Delta T_{sc}}{\Delta T_{sat}} > 6.3 \times 10^4 \text{Bo}^{1.25} \]  
(13)

The regimes of high and low subcooling are shown graphically in Figure 1. The complete correlation is shown graphically in Figure 2.

For \( Re_c > 2,300 \), \( h_c \) is calculated by the Dittus-Boelter equation:

\[ h_c = 0.023 (GD_a)^{0.8} P_{t, s}^{0.4} k_f D \]  
(14)

All properties are calculated at the bulk liquid temperature. The equivalent diameter \( D_e \) for the circular tube is the inside tube diameter. For annuli, \( D_e \) is defined as follows:

\[ \delta > 4 \text{ mm}, \quad D_e = \frac{4 \times \text{flow area}}{\text{wetted perimeter}} \]  
(15)

\[ \delta < 4 \text{ mm}, \quad D_e = \frac{4 \times \text{flow area}}{\text{heated perimeter}} \]  
(16)

Table 1

<table>
<thead>
<tr>
<th>Complete Range of Parameters of Experimental Data with Which the Shah Correlation [9, 10] Has Been Verified</th>
</tr>
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<tbody>
<tr>
<td>Geometry</td>
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<tr>
<td>Heat transfer, K</td>
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<td>Fluids</td>
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<tr>
<td>( \psi )</td>
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<tr>
<td>( \Delta T_{sc} / \Delta T_{sat} )</td>
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<tr>
<td>( \text{Re}_c )</td>
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<tr>
<td>( \text{G}, \text{kg/m}^2 )</td>
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<tr>
<td>( \text{q}, \text{W/m}^2 )</td>
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<td>( \text{Re}_c \times 10^{-7} )</td>
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<tr>
<td>( \text{Bo} \times 10^3 )</td>
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<td>( P_r )</td>
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<td>( P_r )</td>
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\( \delta \) is the gap between the two tubes forming the annulus. All the data analyzed were for \( Re_\delta > 2,300 \) except one data set for an annulus which was for \( Re_\delta \) down to 1,400. This data set was better correlated when \( h \) was calculated by a laminar flow equation.

The complete range of data analyzed is given in Table 1. For the approximately 900 data points analyzed, the mean deviation of \( h \) was found to be about 9\% with over 97\% of the data points within \( \pm 30\% \). The two-phase heat transfer coefficients \( h \) is defined as:

\[
h = \frac{q}{(T_w - T_b)}
\]  

(17)

\[ \Delta T_{SC} / \Delta T_{SAT} \]

**Figure 1.** Demarcation between the regimes of high and low subcooled boiling according to the correlation of Shah [9] for boiling inside tubes and annuli.

\[ \Delta T_{SC} / \Delta T_{SAT} \]

**Figure 2.** The correlation of Shah [9] for subcooled boiling inside tubes and annuli in graphical form.
Thus the accuracy of the Shah correlation for boiling heat transfer is comparable to that of correlations for single-phase heat transfer. The data analyzed are for a variety of heating surfaces and fluids and cover a very wide range of data. Hence it appears that this correlation can be applied to any commercially manufactured tube and a wide variety of fluids.

Techniques Based On Pool Boiling Correlations

Several predictive techniques have been proposed in which forced-convection heat transfer is calculated by superposing pool boiling and single-phase heat transfer. The best known among these is the correlation of Rohsenow [11] according to which:

\[ q = q_{wp} + q_{sc} \]  

\[ h_{wp} \Delta T_{wp} = h_{wb} \Delta T_{sat} + h_{lb} \Delta T_{lb} \]  

\[ h_{wb} \] is calculated by a correlation for pool boiling and \( h_{lb} \) by a single-phase heat transfer correlation such as the Dittus-Boelter equation. Thus it is assumed that forced convection does not affect pool boiling. Rohsenow used his own pool boiling correlation [12] to calculate \( h_{wb} \). While Rohsenow was able to correlate several data sets, this agreement is not fully convincing as the value of factor \( C_d \) in his pool boiling correlation could be adjusted to suit the data. The values of \( C_d \) ranged from 0.0022 to 0.013. Bergles and Rohsenow [1] conducted tests in which forced convection and pool boiling tests were done on the same tube. Their measurements indicated that forced convection boiling could not be considered an extrapolation of pool boiling. They recommended that the forced-convection boiling curve be determined from forced-convection data, not pool boiling data. The Shah correlation is in accordance with this recommendation.

Another superposition technique has been proposed by Kutateladze [13]. It is expressed by the following equation:

\[ h_{wfb} = \left[ 1 + \left( \frac{h_{wfb}}{h_{lb}} \right)^{n-1} \right] h_{lb} \]  

The exponent \( n = 2 \). While the Rohsenow correlation assumes that the nucleate boiling and single-phase convection effects are independent and additive, the Kutateladze approach assumes some suppression of nucleate boiling due to forced-convection effects. At \( n = 1 \), there is no suppression and the Rohsenow correlation is obtained. The increasing values of \( n \) represent increasing bubble suppression.

In the correlation of Chen [14] for saturated boiling, nucleate boiling component is calculated by a pool boiling correlation and then multiplied by a suppression factor which depends on two-phase Reynolds number. Collier [58] reports that it has been extended to subcooled boiling as follows:

\[ q = h_{wb} \Delta T_{sat} + h_{lb} \Delta T_{lb} \]  

\( h_{wb} \) is calculated by the Chen correlation with the factor \( F_c = 1 \). Satisfactory agreement with subcooled boiling data for water, n-butyl alcohol, and ammonia is reported but no details are given.

Various Predictive Techniques

A dimensionless correlation presented by Hodgson [8] is as follows:

\[ h_{wfb} = 141 \, B_0^{0.87} \left[ \frac{\Delta T_{sw}}{h_{lb}} \right]^{-0.55} \left[ \frac{Z_{lb}}{X_{cp}} \right]^{-0.85} \times h_{lb} \]
where:

\[ Z = -\frac{d(p/p)}{dp} \]

(23)

\[ X = dT_{sat}/dp \]

(24)

All properties are evaluated at the bulk fluid temperature. Satisfactory agreement was found with their own data for water and the data of Noel [15] for ammonia.

Numerous dimensional equations for water have been proposed. Among these, the best known is the following dimensional equation of Jens and Lottes [16]:

\[ \Delta T_{sat} = 25q^{0.85}p^{-0.82} \]

(25)

with \( \Delta T_{sat} \) in °C, \( q \) in MW/m², and \( p \) in bar (absolute). This equation was based on data for water flowing upward in nickel and stainless steel tubes of diameter 3.63 to 5.74 mm. Pressure varied from 7 to 172 bar, water temperature from 115° to 340°C, mass velocity from 11 to \( 1.04 \times 10^6 \) kg/m²s and heat flux up to 12.5 MW/m².

The methods proposed by Bergles and Rohsenow [1] is an extension of the Kutateladze method [13]. The total heat flux \( q \) is determined as follows:

\[ q = q_{spc} \left( 1 + \left[ \frac{q_{sb}}{q_{spc}} \left( 1 + \frac{q_{sb}}{q_{spc}} \right) \right]^{1/2} \right) \]

(26)

\( q_{sb} \) is the heat flux for initiation of boiling which may be calculated for water by Equation 2. \( q_{spc} \) is the heat flux for fully developed nucleate boiling. It is to be calculated from data obtained with forced convection, for example the Jens-Lottes correlation. This method can be applied only if some forced convection boiling data (or correlation) are available for the liquid in consideration.

Percupile et al. [17] have given the following dimensional equation:

\[ \Delta T_{sat} = 2.28(p/p_0)^{-0.27}q^{0.3}D^{1.4} \]

(27)

\( \Delta T_{sat} \) is in °C, \( q \) in W/m², and \( D \) in m. This equation is based on their data for R-11, R-12, and R-113 flowing up through vertical tubes of 6-mm and 18.8-mm ID, pressure from 1.43 to 9.64 bar, \( q \) from 14.7 to 102 kW/m², and vapor quality from -0.06 to 0.21.

**Design Recommendations**

1. For prediction of boiling inception the correlation of Frost and Drakovic, Equation 4, is recommended. It should be realized that boiling inception depends on so many factors that no predictive technique can be fully relied upon.

2. For the calculation of heat transfer, the correlation of Shah is recommended for \( Re_p > 2,300 \). At lower Reynolds numbers, use the Rohsenow correlation Equation 19.

**SINGLE CYLINDER WITH CROSSFLOW**

Many heat exchangers, notably kettle reboilers and shell-and-tube evaporators, involve crossflow over tube bundles. For prediction of heat transfer in such tube bundles, it is desirable to be able to predict heat transfer to a single tube. See, for example, the computational method of Brisbane et al. [18]. For this reason, several experimental studies on heat transfer to single cylinders have been done and attempts made to develop predictive techniques. Some of these predictive techniques are now discussed.
Shah Correlation

Shah [19] has given a correlation which shows satisfactory agreement with available experimental data. The correlating parameters are the same as in his correlation for flow through tubes, presented earlier. The two correlations are similar but differ quantitatively.

At zero subcooling, the following equations apply:

\[
\text{Bo} > 2.5 \times 10^{-4}, \quad \Psi_0 = 443 \text{Bo}^{0.41} \\
\text{Bo} < 2.5 \times 10^{-4}, \quad \Psi_0 = 19 \text{Bo}^{0.23}
\]  

(28)  

(29)

If Equation 29 predicts \( \Psi_0 < 1 \), use \( \Psi_0 = 1 \).

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Figure 3. Demarcation between the regimes of high and low subcooled boiling regimes according to the correlation of Shah [19] for boiling with forced convection across cylinders.

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Figure 4. Comparison of the Shah correlation [19] with data for water boiling during flow across single cylinders.
There are two regimes of subcooling, namely the high subcooling regime and the low subcooling regime. In the low subcooling regime:

\[ \Psi = \Psi_0 \]  \hspace{1cm} (30)

In the high subcooling regime:

\[ \Psi = \Psi_0 + \Delta T_{SC}/\Delta T_{SAT} \]  \hspace{1cm} (31)

The demarcation between the two regimes of subcooling is shown graphically in Figure 3. It is seen that if \( \Delta T_{SC}/\Delta T_{SAT} > 4 \), the regime is that of high subcooling. The regime is that of high subcooling also when \( Bo < 5.4 \times 10^{-4} \) and the following inequality is satisfied:

\[ \Delta T_{SC}/\Delta T_{SAT} > 7.63 \times 10^4 Bo^{1.31} \]  \hspace{1cm} (32)

For calculating the single-phase heat transfer, best results were found with the following equation:

\[ \frac{h_D}{k'} = 0.21 \left( \frac{GD}{\mu'} \right)^{0.4} Pr_{eff}^{0.4} \]  \hspace{1cm} (33)

All properties are calculated at the bulk fluid temperature. The mass velocity \( G \) is based on the minimum flow area, i.e., the clearance between the test cylinder and the flow channel in which it is located. The value of \( G \) used in calculating \( Bo \) in Equation 33 is the same as was used in Equation 5. Shah obtained Equation 33 by analyzing data for single-phase heat transfer.

The arguments used in developing this correlation are the same as those used in developing the Shah correlation for boiling in tubes, as described in the previous Section.
Shah compared this correlation with data from ten experimental studies. These were the only data in open literature. The range of parameters covered by these studies is listed in Table 2. He found that the use of Equation 33 gives satisfactory results with all data except those of Yilmaz [20] which are overpredicted. The Yilmaz data give satisfactory agreement when \( h_c \) is calculated by the following equation of Fand and Keshwani [21]:

\[
N_u = (0.255 + 0.699 \text{Re}^{0.3} \text{Pr}^{0.3})
\]

(34)

All the properties in this equation are calculated at the film temperature. The Yilmaz data were analyzed using this equation while all other data were analyzed using Equation 33. The comparison between the data and correlation is shown in Table 2 and Figure 4. The heat transfer coefficients in all 334 data points were predicted with a mean deviation of 9.5%, with only 6 data points showing deviations greater than 30%. Thus the accuracy of the correlation is comparable to correlations for single-phase heat transfer. Better accuracy, though always desirable, will be difficult to achieve.

The data include four different tube materials and three fluids. Furthermore, the correlating parameters are the same as in the Shah correlation for boiling inside tubes which has been verified with many fluids and tube materials. Hence this correlation may be expected to be generally applicable to all common fluids and commercially prepared surfaces. The range of other parameters is very wide except that the reduced pressure ranged from 0.005 to 0.05. Application to higher pressures should be made with caution. As the shell side of heat exchangers is rarely operated at high pressures because of structural considerations, the verified pressure range covers most practical needs.

It must be remembered that this correlation is for forced convection boiling. At \( G = 0 \), heat transfer occurs by natural convection and pool boiling correlations apply. Hence the Shah correlation should not be used at velocities lower than those for which it has been verified.

**Other Predictive Techniques**

Several researchers have compared their measurements with the Rohsenow superposition correlation, Equation 19. Yilmaz and Westwater [22] and Leppert et al. [23] found satisfactory correlations by this method, using their own pool boiling data to calculate \( q_{p} \). McKee [25] using his own pool boiling, found that some of his data were satisfactorily correlated while some were under-predicted. Fand et al. [21] and Lemmert and Chawla [24], state that their data could not be correlated in terms of Equation 19. Hence the Rohsenow method has not been successful even where pool boiling data for the particular test section were available. The difficulties in accurately calculating pool boiling heat transfer are well-known.

Lemmert and Chawla [24] compared their data with the Kutateladze superposition correlation, Equation 20. Poor agreement was found. Fand et al. [21] found this equation unsatisfactory with \( n = 2 \), the value given by Kutateladze. However, their data were well-correlated with \( n = 5.5 \).

McKee [25] attempt to improve the Rohsenow method by distinguishing between areas with bubble nucleation and those without bubble nucleation. His equation is:

\[
qA = q_{pc}A_{bc} + q_{pc}(A - A_{bc})
\]

(35)

\( A_{bc} \) is the tube surface area with bubble nucleation and \( A \) is the total surface area. He determined \( A_{bc}/A \) by a dimensional equation which fitted his data. Whether it fits other data, is unknown.

Lemmert and Chawla [24] also used the approach of Equation 35 but developed different expressions for determining \( A_{bc}/A \) and heat flux due to bubble nucleation. While their expressions fit their own data well, they have not been compared with any other data.

From the previous discussions, it is clear that none of these predictive techniques can be used with confidence.
Design Recommendations

Use the Shah correlation, Equations 30 and 31, with \( h \) calculated by Equation 33. If single-phase heat transfer data are available, adjust the constant in Equation 33 to fit them. The verified range of this correlation is given in Table 2. Use outside this range is not recommended. It is especially important that the correlation be not applied to \( u, G \), and \( \text{Re}_t \) lower than the minimum listed in Table 2.

CRYOGENIC FLUIDS

Cryogenic fluids are Newtonian and their Prandtl numbers are comparable to that of water. Hence one would expect their behavior to be similar to that of ordinary fluids. Only two sets of analyzable data for subcooled boiling of cryogenic fluids are available. These are the data of Pappel and Hendricks [32] and Klimenko and Sudarchikov [33], both for nitrogen in electrically heated tubes. The present author found that data from both of these sources are in satisfactory agreement with the correlation of Shah presented earlier. Hence the available experimental data for subcooled boiling support the view that heat transfer to cryogenic fluids is not different from that of noncryogenic, nonmetallic fluids. However, there is some indication of differences which are now discussed.

One possible difference is in the influence of thermophysical properties of the heating surface. Stephan and Abdel-Salam [34] developed correlations for pool boiling heat transfer. They found that for cryogenic fluids,

\[
h_\text{p} \propto S^{0.77}
\]

where

\[
S = \frac{(kT_c)}{(k\rho c_p)} \text{ of heating surface}
\]

For noncryogenic fluids, they found no influence of \( S \). This suggests that \( S \) may also have an influence on forced-convection boiling of cryogenic fluids.

Klimenko [35] analyzed the data for saturated boiling of nitrogen, neon, and hydrogen in tubes. He presented a correlation according to which:

\[
h_\text{p} \propto S^{0.2}
\]

Shah [36] compared data for saturated boiling of helium, hydrogen, nitrogen, and argon in tubes with the correlations of Shah [37, 38] and Klimenko [35]. The Shah correlation has no factor for surface properties, and at zero vapor quality it reduces to Equation 7. Data were from 11 sources. Data from 9 sources showed adequate agreement with the Shah correlation. Data of Steiner et al. [39, 40] and Mohr and Runge [41] for nitrogen and neon, respectively, were underpredicted by a factor of up to 3. However, the deviations were uncorrelated to \( S \). These data showed satisfactory agreement if \( h_\text{p} \) at zero quality was calculated by the superposition correlations of Rohsenow or Kuntz and with \( h_\text{p} \) by the Stephan-Abdel-Salam correlation. However, this procedure resulted in overprediction of other data. The Klimenko correlation showed satisfactory agreement with only 3 of 11 data sets and the deviations were not related to \( S \).

Hence, the evidence from the experimental data from both saturated and subcooled boiling studies do not support the view that heat transfer to cryogenic fluid is influenced by data from two sources are much higher than the Shah correlation and indicate little or no suppression of bubble nucleation by forced-convection. This anomaly remains explained.

Another difference is in hysteresis. Strong hysteresis has been reported during saturated boiling of helium [42, 43] and to a lesser extent in saturated boiling of nitrogen [39, 44] and neon [41]. Ordinary fluids show no hysteresis during saturated boiling and thus this is a
significant difference between the behaviour of cryogenic and noncryogenic fluids. In the data of Ogata and Sato [42] for saturated boiling of helium in tubes, the heat transfer coefficients with decreasing heat flux were up to four times higher than those with increasing heat flux. Jergel et al. [43] also noted strong hysteresis during saturated boiling of helium in rectangular channels. Hence one would expect strong hysteresis during subcooled boiling of helium. Klein [44] and Steiner and Schlünder [39] have reported hysteresis during saturated boiling of nitrogen in tubes but the difference between the higher and lower boiling curves did not exceed 20%. Similar results have been reported by Mohr and Runge [41] for saturated boiling of neon in tubes. Hence some hysteresis is likely in subcooled boiling of nitrogen and neon but is likely to be much less than with helium. It should however be noted that Klimenko and Sudarchekov [33] found no hysteresis during subcooled boiling of nitrogen and Papell and Hendricks [32] have made no mention of hysteresis during their experiments with subcooled boiling of nitrogen and neon.

Papell and Hendricks Correlations

Subcooled boiling of cryogenic fluids was the subject of studies by Papell and Hendricks [32]. In that study, nitrogen and neon flowed up through an electrically heated inconel tube of 19.8 mm ID. The parameters in nitrogen tests were: \(p = 11.1\) to 28.7 bar, flow rate 2.7 to 6.5 g/s, \(q = 4.1\) to 21.1 W/cm², and subcooling up to 26.5°K. The range of neon data was: \(p = 11.2\) to 16.6 bar, flow rate from 4 to 10.8 g/s, \(q = 1.8\) to 20.5 W/cm², and subcooling up to 11.6°K. All data were correlated within ± 30% by the following equation:

\[
\frac{h_{nf}}{h_L} = 100 \left[B_0 \left(\frac{\rho_L}{\rho_L^{sat}}\left(\frac{T_L}{T_L^{sat}}\right)^{12} \left(\frac{\rho_a}{\rho_a^{sat}}\right)^{14}\right)^{\gamma}ight]
\]  

(39)

where

- \(i\) = the enthalpy of liquid at actual temperature
- \(i^{sat}\) = the enthalpy of saturated liquid
- \(\rho_L\) = the density of liquid at bulk temperature

It should be noted that nitrogen data from this study show satisfactory agreement with the correlations of Shah [9] and Klimenko [35]. The data for neon were not given in an analyzable form.

Papell and Hendricks correlated the data for boiling inception by the following relation:

\[
B_0 \left(\frac{\rho_L}{\rho_L}\right) \left(\frac{\rho_a^{sat}}{\rho_a}\right)^{14} \left(\frac{1}{i_L - i^{sat}}\right)^{12} = 0.0014
\]

(40)

Design Recommendations

1. The correlation of Shah given in the section on “Heat Transfer during Boiling in Channels” is the best choice for calculating heat transfer in tubes and annuli. It should however be noted that some data are much higher than this correlation.
2. Very strong hysteresis may be expected with helium. It is expected that the Shah correlation will yield the lower boiling curve.
3. For inception of boiling, recommendations are the same as in the section mentioned in Recommendation 1. Equation 40 may be used in the range of data on which it is based.

LIQUID METALS

Boiling of liquid metals has been found to be considerably different from that of ordinary liquid. Correlations of heat transfer to nonmetallic fluids are usually inapplicable to...
metallic fluids. Ordinary liquids generally show incipient superheats of only a few degrees. In liquid metals superheats of over 100°K have been measured. Large fluctuations in wall temperatures and fluctuations in fluid have often been experienced.

**Boiling Inception And Instabilities**

Several studies on potassium have shown that when liquid enters the heating tube in a subcooled state, boiling inception superheat is high and large fluctuations in wall temperatures occur. For example, Aladzev [45] reports the following typical behavior. Wall temperature first increases with time. When a certain value (greater than \( T_{\text{sat}} \)) is reached, rapid boiling of potassium takes place and the wall temperature drops to \( T_{\text{sat}} \) at the rate of 20° to 25°C/second. The process is then repeated. As the pressure increases, the superheat decreases, while the frequency increases. The observations of Hoffman and Krakoviak [46] are similar. With a plain stainless steel tube, they found superheats in excess of 160°C and wall temperature fluctuations at 0.025 Hertz. The superheat decreased sharply when the tube surface was provided with artificial nucleation sites. In the experiments of Noyes and Lurie [47] with sodium in an annulus, wall temperature fluctuations were also observed under subcooled conditions, but their amplitude and frequency were not as high.

Chen [48] measured incipient superheats in subcooled potassium at pressures between 0.55 and 0.96 bar flowing in tubes. Incipient superheats between 9° and 65°K were measured and those were found to be related to deactivation conditions preceding boiling tests and boiling pressure. Chen points out that analyses based on the Bergles-Roshenov model, discussed earlier, will underpredict the incipient superheats for metals. This is because the cavity size corresponding to predicted inception conditions is so large that such cavities are likely to be flooded by liquid and hence inactive. As a consequence boiling will start only when superheat has increased sufficiently to activate smaller cavities containing trapped gases. Chen developed an analytical model which agrees with his own data.

Dodonov et al. [49] carried out an extensive study of incipient boiling superheat of potassium in straight and coiled tubes. The straight tubes were 6- to 6.1-mm ID stainless steel with their inner surface reamed on a lathe. The coiled section was made from a 10-mm ID stainless steel tube. The coil diameter was 162 mm, and coil pitch was 73 mm. The range of variables was: \( p \) from 0.6 to 3.6 bar, \( G \) from 0 to 277 kg/m²s, \( q \) from 32 to 300 \( \times 10^3 \) W/m², and inlet subcooling from 43° to 488°K. It was found that incipient boiling superheat was independent of \( G \), \( q \), and inlet subcooling. The unsteadiness in approach to boiling was not found to affect superheat. The superheat at the tube exit prior to vapor generation and the superheat in the center of tube with vapor generation were found to be the same. Superheats measured at different times and with different prior histories were found to be essentially the same. This contradicts the observations of Chen [47] discussed earlier. Superheat was found to depend on pressure. Their data were correlated by the following dimensional equation:

\[
\Delta T_{\text{b}} = 100 \exp (-50 p) \tag{41}
\]

\( \Delta T_{\text{b}} \) is in °K. This equation was found to also show reasonable agreement with data from several other sources.

Peppler [50] has reported experiments on incipient boiling superheat for flow of sodium in single channels and pin bundles. He states that every time boiling occurred in a new test section, no measurable superheat was observed. After repeated boiling under heat fluxes greater than 150 W/cm², the maximum superheat found was 25°C. This increase in incipient superheat may perhaps be explained in terms of surface deterioration caused by corrosion, as suggested by Schultheiss [51]. Peppler further reports that in boiling tests with pin bundles without gas injection, 5 Hz pulsations in outlet velocity were found. With gas injection, the pulsations were less evident.

Henry et al. [52] measured incipient superheats for sodium in an annulus 9.5-mm OD, 5.84-mm ID. Pressures varied from 0.14 to 0.68 bar. Superheats from 0° to 75°K were
measured. The data were found to be consistent with the reentrant cavity model when history and gas solubility were properly accounted for.

Schmucker and Grigull [53] carried out tests on boiling of mercury in a tube. Boiling was found to be intermittent and the behaviour was generally similar to that of potassium described by Aladyev [45].

Tang et al. [54] conducted tests on potassium-mercury mixtures in tubes. Tests included transition from single-phase to subcooled boiling. They report no significant incipient superheats or instabilities. The tests of Mori et al. [55] on mixtures of mercury, bismuth, and lead also included transition from single-phase flow to subcooled boiling. They also do not report any significant incipient superheats or flow-temperature fluctuations.

Summarizing, boiling of pure subcooled liquid metals is often unsteady. Large amplitude cyclic variations in wall temperatures often occur. The variations are generally stronger for potassium than sodium. Mercury also shows behavior similar to potassium. Large incipient superheats have often been found with sodium and potassium. On the other hand some tests on sodium have shown negligible incipient superheat. Some data show incipient superheat to be related to previous history while some do not show such a relation. Finally mixtures of metals show no incipient superheat and no unsteadiness in boiling. Clearly, experimental evidence is conflicting and no method of predicting incipient superheat can be relied upon. However, Equation 41 for potassium shows reasonable agreement with a considerable amount of data.

Heat Transfer

Noyes and Lurie [47] studied subcooled boiling of sodium in an electrically heated annulus. Boiling heat transfer coefficients were found to be only slightly higher than those for single-phase forced convection.

Tang et al. [54] measured heat transfer to potassium-mercury mixtures in a 7.75-mm ID stainless steel tube. Pressure varied from 0.1 to 0.9 bar and subcooling was up to 167°K. Boiling heat transfer coefficients were found to be only slightly higher than those for single-phase convection.

Experiments on forced-convection boiling of rubidium and cesium have been reported by Fisher et al. [56, 57]. The tests were done in 7.1-mm ID tubes made of columbium-zirconium alloy. The range of parameters investigated for rubidium was: \( G = 158 \) to \( 1192 \) kg/m²s, \( T_{sat} = 977 \) to 1255°K, \( q = 0 \) to 103 kW/m², inlet subcooling = 111°C to 222°C, and outlet quality from 0 to 0.8. The data for the nucleate boiling of rubidium were correlated by the dimensional equation:

\[
h_{fg} = a q^b
\]  

(42)

<table>
<thead>
<tr>
<th>Temp. range (°F)</th>
<th>n</th>
<th>\log ( q^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,350-1,400</td>
<td>1.126</td>
<td>-2.046</td>
</tr>
<tr>
<td>1,400-1,450</td>
<td>0.917</td>
<td>-0.918</td>
</tr>
<tr>
<td>1,450-1,500</td>
<td>1.181</td>
<td>-2.294</td>
</tr>
<tr>
<td>1,500-1,550</td>
<td>1.325</td>
<td>-3.044</td>
</tr>
<tr>
<td>1,550-1,600</td>
<td>1.405</td>
<td>-3.414</td>
</tr>
<tr>
<td>1,600-1,650</td>
<td>1.333</td>
<td>-3.328</td>
</tr>
<tr>
<td>1,650-1,700</td>
<td>0.558</td>
<td>-1.268</td>
</tr>
<tr>
<td>1,700-1,750</td>
<td>1.528</td>
<td>-4.427</td>
</tr>
</tbody>
</table>
forced-convection subcooled boiling

$ h_r $ is in Btu/hr ft² °F and $ q $ in Btu/hr ft². The value of $ a $ and $ n $ were determined at 50°F intervals and are given in Table 3.

**Design Recommendations**

1. No reliable method for calculating incipient superheat is available. For potassium, Equation 41 may be used in its verified range. For mixtures of mercury with other metals, incipient superheats is negligible.

2. No verified method for predicting the heat transfer rate is available. If no experimental data are available, the following conservative approach is suggested. Calculate pool boiling and single-phase-convection heat transfer coefficients separately. Use the larger of the two.

**MULTICOMPONENT FLUIDS**

In discussing the boiling of multicomponent mixtures, one must first distinguish between azeotropic and nonazeotropic mixtures. In azeotropic mixtures, the concentrations of components in the vapor and liquid phases are the same and remain the same during evaporation. Such mixtures behave like single-component fluids. Heat transfer in azeotropic mixtures can be calculated by the methods for single-component fluids, using the mean mixture thermophysical properties. Behaviour of nonazeotropic mixtures is much more complex and is the subject of this section.

Fink et al. [29] studied boiling of mixtures of R-11 and R-113 during forced flow across a heated cylinder. Heat transfer coefficients for the mixtures were always lower than what will be expected from linear interpolation of heat transfer coefficients for the pure fluids in proportion to concentration. Similar results have been reported during pool boiling of multicomponent mixtures. The explanation is that the low boiling point component evaporates faster with the result that its concentration in the liquid surrounding the bubble is less than in the bulk of liquid. Thus the boiling point of the liquid surrounding the bubble is higher than that of the bulk liquid. As a consequence, the temperature difference causing bubble nucleation is lower than in the case of pure fluids.

Data for forced-convection subcooled boiling are scarce and no verified method for predicting heat transfer is available. For rough estimates, the author's recommendation is to use the Kutateladze superposition method (Equation 20) with $ h_{ns} $ calculated by a correlation suitable for pool boiling of multicomponents mixtures. The single-phase heat transfer coefficients $ h_r $ is to be calculated using mixture mean properties. This recommendation is only for nonmetallic fluids. Recommendations for metallic fluids are given earlier.

**NOTATION**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ A $</td>
<td>area (m²)</td>
</tr>
<tr>
<td>$ B_{0} $</td>
<td>boiling number $ = q/(Gt_{w}) $, dimensionless</td>
</tr>
<tr>
<td>$ C_p $</td>
<td>specific heat at constant pressure $ (J/kg\cdot K) $</td>
</tr>
<tr>
<td>$ C_{p_r} $</td>
<td>specific heat of liquid at constant pressure $ (J/kg\cdot K) $</td>
</tr>
<tr>
<td>$ D $</td>
<td>diameter (m); when flow is inside the tube, $ D $ is the inside diameter; when flow is outside the tube, $ D $ is the outside diameter</td>
</tr>
<tr>
<td>$ D_{eq} $</td>
<td>equivalent diameter of channel (m)</td>
</tr>
<tr>
<td>$ D_i $</td>
<td>outside diameter of the inner tube of an annulus (m)</td>
</tr>
<tr>
<td>$ D_o $</td>
<td>inside diameter of the outside tube of an annulus (m)</td>
</tr>
<tr>
<td>$ g $</td>
<td>acceleration due to gravity (m/s²)</td>
</tr>
<tr>
<td>$ G $</td>
<td>total mass flux (kg/m²s)</td>
</tr>
<tr>
<td>$ h $</td>
<td>heat transfer coefficient (W/m²K)</td>
</tr>
</tbody>
</table>
HEAT TRANSFER MECHANISMS

\( h_s \) heat transfer coefficient in single-phase flow assuming all mass (vapor + liquid) flowing in liquid form (W/m²·K)
\( h_{nb} \) heat transfer coefficient due to nucleate boiling (W/m²·K)
\( h_{pb} \) heat transfer coefficient calculated by a pool boiling correlation (W/m²·K)
\( h_{TP} \) two-phase heat transfer coefficient = \( h_s (T_s - T_0) \) (W/m²·K)
\( i \) enthalpy of liquid (J/kg)
\( i_s \) enthalpy of saturated liquid (J/kg)
\( i_l \) latent heat of vaporization (J/kg)
\( k \) thermal conductivity (W/m·K)
\( Nu \) Nusselt number = \( h D/k_s \), dimensionless
\( p \) pressure (N/m²)
\( p_c \) critical pressure (N/m²)
\( p_r \) reduced pressure = \( p/p_c \), dimensionless
\( Pr \) Prandtl number of liquid = \( C_p u_s /k_s \), dimensionless
\( q \) heat flux (W/m²)
\( q_{bo} \) heat flux at boiling inception (W/m²)
\( q_{nb} \) heat flux due to nucleate boiling (W/m²)
\( q_{pb} \) heat flux assuming pool boiling (W/m²)
\( q_{BC} \) heat flux due to single-phase convection (W/m²)
\( r \) radius of bubble (m)
\( R_e \) Reynolds number = \( GD/\mu_s \), dimensionless
\( S \) parameter defined by Equation 37, dimensionless
\( T \) bulk liquid temperature (°K)
\( T_{sat} \) saturation temperature (°K)
\( \Delta T_{3c} \) = \( T_{sat} - T_b \) (°K)
\( \Delta T_b \) = \( T_s - T_b \) (°K)
\( \Delta T_{sat} \) = \( T_s - T_{sat} \) (°K)
\( \Delta T_{sat} \) at boiling inception (°K)
\( \nu_\text{sh} \) specific volume of vapor minus specific volume of liquid both at saturation (m³/kg)

Greek Symbols

\( \Psi \) = \( h_{TP}/h_s \), dimensionless
\( \Psi_b \) value of \( \Psi \) at zero subcooling and zero quality
\( \rho \) density (kg/m³)
\( \mu \) dynamic viscosity (Ns/m²)
\( \sigma \) surface tension (N/m)
\( \delta \) annular gap = \( (D_0 - D_i)/2 \) (m)

Subscripts

\( b \) bulk (i.e. mixing cup condition)
\( f \) with properties at film temperature = \( (T_s + T_b)/2 \)
\( g \) of vapor
\( \ell \) of liquid
\( \text{SAT} \) at saturation temperature
\( \text{w} \) at wall

Abbreviations

ID inside diameter
OD outside diameter
SS stainless steel

REFERENCES


